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CONDUCTIVE THERMAL CONDUCTIVITY OF FIBROUS MATERIALS UNDER TRANSIENT GAS FLOW

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UDC 536.2.01

The results are reported from computational studies of the conductive thermal conductivity of fibrous materials working at pressures correspnding to transient gas flow in pores.

Energy transfer through a layer of fibrous material is determined by such elementary processes as thermal conductivity through a framework and a gas, radiative heat transfer, and forced and free convection. The intensity of these processes depends in many ways on the external conditions [1]. The region of application of highly porous thermal insulation materials is characterized by variation of the pressure from hundreds of atmospheres to space vacuum, a temperature range from 20 to 2000 K, and different compositions of the gas filling the pore. The contribution of one elementary process or another to the overall heat transfer process will vary, depending on the parameters indicated above. Thus, radiative heat transfer is virtually absent at low temperatures and the effect of free convection is negligible at pressures $P < 100$ kPa while its role is decisive at pressures of the order of 100 MPa [1]. Of great interest is the range of pressures where transient gas flow conditions in a porous medium are realized. A number of applications of heat-insulation coatings typically have the pressure-time characteristic shown in Fig. 1. In the indicated pressure range heat transfer in the pores is determined by the interaction of gas molecules with each individual fiber (a similar situation arises when radiation interacts with the elements of a fibrous framework).

Analysis of the mathematical models in [1, 2] for heat transfer through the gas and framework permit the conclusion that for structures formed by fibers with different diameters, lengths, orientations, and thermophysical characteristics (TPC's) the thermal conductivity will depend on the anomalies in the fiber distribution according to size, orientation, and thermophysical properties as well as on such integrated indices as porosity and average fiber diameter. The effect of the distribution on the TPC's can manifest itself most in transient pressure regimes.

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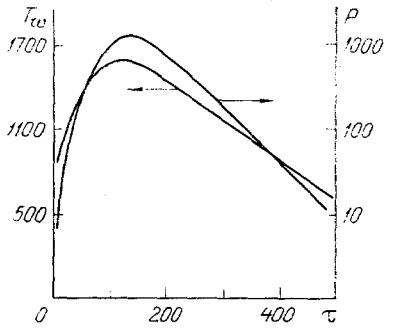

Fig. 1. Characteristic time dependence of the pressure and temperature at the frontal surface of a heat-protective coating. T_w, K; P, N/m^2; τ, sec.

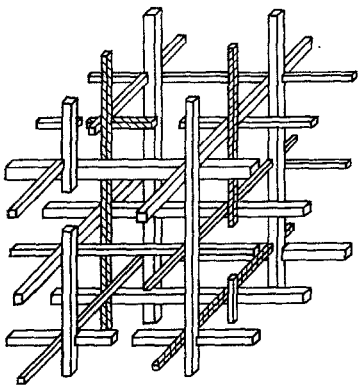

Fig. 2. Model structure.

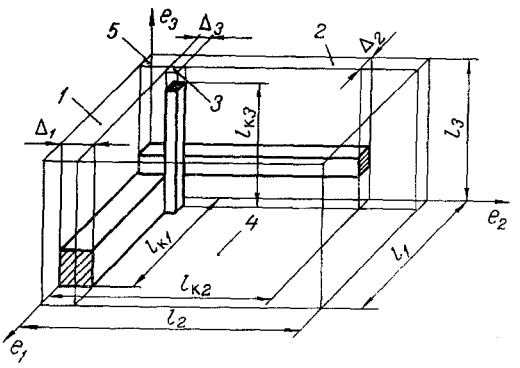

Fig. 3. Elementary volume.

The foregoing discussion calls for a more profound analysis of the methods of calculating the conductive thermal conductivity of highly porous fibrous system as applied to the indicated pressure range.

During the development of mathematical models of the conductive thermal conductivity of fibrous materials a transition is made, as a rule, from the real structure of the material to some model structure which would be an adequate initial structure and at the same time would lend itself to mathematical investigation. In the general case fibrous materials can be classified as randomly inhomogeneous media, since the parameters of the constituent elements of the structure (diameters, lengths, orientations, thermophysical properties) are random quantities. A randomly inhomogeneous fibrous structure with orthogonally arranged fibers (Fig. 2) is proposed for modeling fibrous composites; the fibers are assumed to have the same distribution with respect to orientations and TPC's. The anisotropy indices a_i ($i = \overline{1, 3}$) of the model structure can be determined on the basis of information about the orientation distribution of the fibers [3]:

$$a_i = \frac{n_i}{n_3}, \quad i = \overline{1, 3},$$

where n_i is the number of fibers per unit volume of the model structure, oriented along the i-th axis. It can be shown that $a_i = \ell_i/\ell_3$, $i = 1, \ldots, 3$, and ℓ_i is the size along the i-th axis of an elementary volume (Fig. 3) isolated in the model structure. Since the anomalies of the structure of fibrous composites affect their mechanical as well as thermophysical properties, it is possible to determine the anisotropy indices a_i ($i = 1, \ldots, 3$), by measuring the tensile strnegth of the material along different axes [4].

The proposed model structure consists of elementary volumes, each of which is characterized by a random vector of the parameters,

$$\xi = (\Delta_i,\ l_i,\ l_{fi},\ l_{fri}\rho_i,\ \lambda_i,\ h_i,\ \Delta_{fri}\ i = \overline{1, 3}),$$

where i is the index of the axis; ℓ_i is the size of the elmentary volume; ℓ_{fi} is the length of the fiber forming the elementary volume; ℓ_{fri} is the length of the element of the fibrous framework in the elementary volume ($\ell_{fri} \leq \ell_i$); Δ_i is the size of the cross section of an element of the fibrous framework; ρ_i and λ_i are the density and thermal conductivity of an element of the fibrous framework; and h_{fri} and Δ_{fri} are the height and cross-sectional size of the contact zone. The probabilistic characteristics of vector ξ depend on the probabilistic characteristics of the constituent elements (fibers) of the structure.

For each elementary volume, using the treatments proposed in [1, 2], we can calculate the values of the density and thermal conductivity, which are random quantities since they depend on vector ξ. If we consider a macrovolume V_n comprising n elementary volumes, however, as the number n of the values of the physical characteristics ρ_n and λ_n of the macrovolume will tend to certain values, which are taken to be the density and thermal conductivity of the fibrous material:

$$\rho_n \xrightarrow[n\to\infty]{} \rho_{f.m.}$$

$$\lambda_n \xrightarrow[n\to\infty]{} \lambda_{f.m.}.$$

Bearing in mind that

$$\rho_n = \sum_{j=1}^{n} \rho_{e.vj}/n,$$

$$\lambda_n = \sum_{j=1}^{n} \lambda_{e.v.j}/n,$$

where $\rho_{e.v.j}$ and $\lambda_{e.v.j}$ are the density and thermal conductivity of the j-th elementary volume, and using the law of large numbers in the Khinchin form [5], we obtain:

$$\rho_n \xrightarrow[n\to\infty]{p} M[\rho_{e.v.j}],$$

$$\lambda_n \xrightarrow[n\to\infty]{p} M[\lambda_{e.v.j}].$$

Thus, henceforth as the density and thermal conductivity of the fibrous material we take the values of the mathematical expectations of these quantities, calculated for an elementary volume.

We must note that vector ξ contains three independent parameters ℓ_1, ℓ_2, ℓ_3, which are important characteristics of the model structure and the elementary volume. These parameters can be determined form the conditions of equality of the apparent density ρ_a of the fibrous material, which can be measured easily, and the apparent density of the model structure. It is assumed that the anisotropy indices a_1 and a_2 of the structure are predetermined.

$$\rho_a = M[\rho_{e.v}(\xi(l_1,\ l_2,\ l_3,\ \ldots))],\ l_1 = a_1 l_3,\ l_2 = a_2 l_3. \tag{1}$$

The solution of problem (1) presumes the existence of a mathematical model of the apparent density of the elementary volume, which has the form

$$\rho_{e.v.} = \sum_{i=1}^{3} (l_{fri}\Delta_i^2 \rho_i) \Big/ \prod_{i=1}^{3} l_i. \tag{2}$$

In order to calculate the mathematical expectations of the density and thermal conductivity of the elementary volume we introduce the space Ω of elementary events, in which we define the vector ξ,

$$\Omega = \{\omega | \Delta_i = x_{ik},\ l_{fi} = y_{il},\ l_{fi} = z_{ip},\ \rho_i = c_{is},\ h_{fri} = f_{ir},$$

$$\Delta_{fri} = t_{im},\ i = \overline{1, 3};\ k = \overline{1, K};\ l = \overline{1, L};\ p = \overline{1, P};$$

$$s = \overline{1, S};\ r = \overline{1, R};\ m = \overline{1, M}\},$$

where S i the number of different materials form which the fibers were made; L and K are the numbers of different values of the fiber lengths and diameters; and M and R are the numbers of different values of the cross-sectional size and height of the contact zone between fibers.

The mathematical expectation of the function of the random vector is determined in a natural manner [5]:

$$M\left[\rho_{\text{e.v}}(\xi(\omega))\right] = \sum_{\omega\in\Omega} \rho_{\text{e.v.}}(\xi(\omega))\, p(\omega),$$
$$M\left[\lambda_{\text{e.v}}(\xi(\omega))\right] = \sum_{\omega\in\Omega} \lambda_{\text{e.v}}(\xi(\omega))\, p(\omega). \tag{3}$$

Bearing in mind that events associated with framework elements oriented along different axes are independent in the aggregate, the probability p(ω) of an elementary event is calculated in terms of the probabilistic characteristics of fibers as follows:

$$p(\omega) = \prod_{i=1}^{3} P(\rho_i = c_{is})\, P(\Delta_i = x_{ik}/\rho_i = c_{is})\, P(l_{\text{f}i} = y_{il}/\Delta_i = x_{ik},$$
$$\rho_i = c_{is})\, P(l_{\text{к}i} = z_{ip}/l_{\text{f}i} = y_{il},\ \Delta_i = x_{ik},\ \rho_i = c_{is}) \times \tag{4}$$
$$\times P(h_{\text{fr}i} = f_{ir},\ \Delta_{\text{fr}i} = t_{im}/l_{\text{fr}i} = z_{ip},\ l_{\text{f}i} = y_{il},\ \Delta_i = x_{ik},\ \rho_i = c_{is}).$$

We note that the random quantity $\ell_{\text{fr}i}$ can assume two values: $z_{1i} = \ell_i$ when a break in a fiber does not fall within the elementary volume and $z_{2i} = \ell_{\text{f}i} - [\ell_{\text{f}i}/\ell_i]\ell_i$ $(z_{2i} < \ell_i)$, when the volume does contain a fiber break. The probabilities that these values will appear are, respectively,

$$P_{1i} = \left[\frac{l_{\text{f}i}}{l_i}\right] \Big/ \left(\left[\frac{l_{\text{f}i}}{l_i}\right] + 1\right),\quad P_{2i} = 1 \Big/ \left(\left[\frac{l_{\text{f}i}}{l_i}\right] + 1\right).$$

To calculate the conductive thermal conductivity of the elementary volume we divide the latter with adiabatic and isothermal planes into fragments (Fig. 3), for which we determine the values of the thermal resistances:

$$R_1 = \frac{\Delta_1}{\lambda_1\Delta_1(l_{\text{fr}\,1} - \Delta_2)} + \frac{l_3 - \Delta_1}{\lambda\,\Delta_1(l_{\text{fr}\,1} - \Delta_2)};$$
$$R_2 = \frac{\Delta_2}{\lambda_2\Delta_2(l_{\text{fr}\,2} - \Delta_1)} + \frac{l_3 - \Delta_2}{\lambda\,\Delta_2(l_{\text{fr}\,2} - \Delta_1)};$$
$$R_3 = \frac{l_{\text{fr}3}}{\Delta_3^2\lambda_3} + \frac{l_3 - l_{\text{fr}3}}{\Delta_3^2\lambda_{\text{p}}} + \frac{h_{\text{fr}}}{\lambda_{\text{fr}}\Delta_{\text{fr}}^2 + \lambda_{\text{pfr}}(\Delta_3^2 - \Delta_{\text{fr}}^2)}; \tag{5}$$
$$R_4 = \frac{l_3}{\lambda_{\text{p}}(l_1l_2 - l_1\Delta_1 - l_2\Delta_2 + \Delta_1\Delta_2 - \Delta_3^2)};$$
$$R_5 = \frac{\Delta_1}{\lambda_1\Delta_1\Delta_2} + \frac{\Delta_2}{\lambda_2\Delta_1\Delta_2} + \frac{l_3 - \Delta_1\Delta_2}{\lambda_{\text{p}}\Delta_1\Delta_2}.$$

The thermal resistance of the elementary volume is calculated on the basis of the circuit diagrams of the thermal resistances

$$R_{\text{e.v.}} = 1 \Big/ \sum_{j=1}^{5} \left(\frac{1}{R_j}\right). \tag{6}$$

The conductive thermal conductivity of the elementary volume is expressed in terms of its thermal resistance

$$\lambda_{\text{e.v.}} = \frac{l_3}{l_1l_2R_{\text{e.v.}}}. \tag{7}$$

Equation (5) includes the thermal conductivity of the pore space, which depends on the structural anomalies of the material as well as on the gas temperature and pressure. The thermal conductivity of the pore space is calculated extensively at present from the Prosolov formula [1, 2]

$$\lambda_p = \frac{\lambda_g^0(T)}{1 + \left[\frac{4c_p/c_v}{(c_p/c_v)+1}\,\frac{(2-A)}{A}\,\Lambda_0 \mathrm{Pr}^{-1}\right] \Big/ (\bar{p}\delta)}, \tag{8}$$

where $\lambda_g^0(T)$ is the thermal conductivity of the gas filling the pores at normal pressure $P_0 = 10^5$ N/m^2; $\bar{p}$ is the relative pressure, $\bar{p} = P/P_0$; δ is the characteristic pore size; c_p and c_v are the gas heat capacities at constant pressure and at constant volume; A is the accommodation coefficient; Λ_0 is the mean-free path of gas molecules at normal pressure; and Pr is the Prandtl criterion.

The thermal conductivity of the pore space is a function of the characteristic pore size δ. It seems incorrect to choose the distance between parallel fibers in an ordered structure [2] as the characteristic pore size. When the random nature of the structure of the material and the processes occurring in the gas are taken into account it seems more correct to take for the characteristic pore size the mathematical expectation of the distance β traveled by a gas molecule between successive collisions with elements of the framework. This very approach is proposed by Bozhkov and Ivanov [6]. Using the relations of the molecular-kinetic theory, they obtained an expressed for δ that holds, however, for a plane fibrous system. Developing this approach, we consider the problem of determining δ for a three-dimensional randomly inhomogeneous structure.

We isolate an elementary layer of thickness b in the material. We consider a random event A, which consists in a gas molecule undergoing collisions in the interval (y, y + b) after traversing a distancy y. The probability P(A) = p of this event depends on b and the structural anomalies of the material and not on y.

Even B consists in a gas molecule traversing a layer (y, y + b) without collision, after previously traveling a distance y:

$$P(B) = P(\bar{A}) = 1 - p = q.$$

Event C consists in a gas molecule traversing a distance y. In this case $P(C) = (P(B))^k$, where k = [y/b]. Event D consists in the distance between successive collisions lying in the interval (y, y + b). Clearly,

$$P(D) = P(A)\,P(C) = (1-q)\,q^k. \tag{9}$$

When we take (9) into account, the probability that a molecular, having traversed the (n − 10-th layer of the thickness b, undergoes collision in the n-th layer is

$$P_n = (1-q)\,q^{n-1}.$$

The mathematical expectation of the random quantity n is

$$M[n] = \sum_{n=1}^{\infty} nP_n = \frac{1}{1-q} = \frac{1}{p}. \tag{10}$$

Suppose that the thickness of an elementary layer is ℓ_3 and the molecule travels along axis e_3 of the model structure. In this case

$$p = M\left[\frac{S_1}{S_2}\right], \tag{11}$$

where S_1 and S_2 are the areas of the projections of elements of the fibrous framework of the elementary volume and of the elementary volume onto the plane $\langle e_1, e_2\rangle$.

Bearing in mind that

$$S_1 = \Delta_3^2 + \Delta_1 l_{\mathrm{fr}1} + \Delta_2 l_{\mathrm{fr}2} - \Delta_1\Delta_2,\ S_2 = l_1 l_2, \tag{12}$$

and taking Eqs. (10)-(12) into account, we determine the characteristic pore size δ, which is equal to the mean free path of a gas molecule between successive collisions with elements of the fibrous framework:

$$\delta = l_3 M[n] = \frac{l_1 l_2 l_3}{M[\Delta_3^2 + \Delta_1 l_{\mathrm{fr}1} + \Delta_2 l_{\mathrm{fr}2} - \Delta_1\Delta_2]}, \tag{13}$$

while for the isotropic determined structure considered in [2] we have:

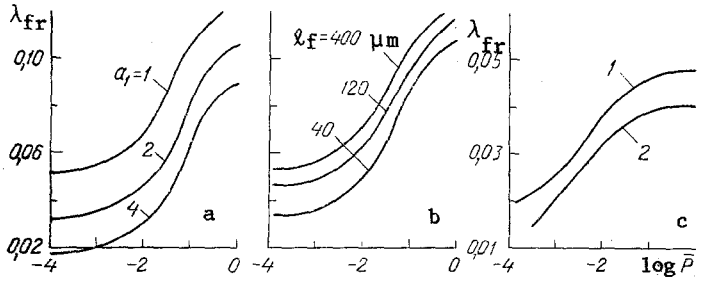

Fig. 4. Effect of the parameters of the structure of a fibrous material on its conductive thermal conductivity: a) effect of the structural anisotropy; b) fiber length; c) fiber distribution according to size. λ_{fr}, W/m·K.

$$\delta = \frac{3\Delta}{2(1-\Pi)},$$

where Π is the porosity of the material, $\Pi = 1 - \rho_{fr}/\rho_f$. This expression differs from the functional relation $\delta(\Pi) = \Delta\sqrt{3/(1-\Pi)}$, which is given in [2] and was in fact obtained for a plane fibrous system.

We can use Eqs. (1)-(8) and (13), therefore, to calculate the conductive thermal conductivity of a fibrous material.

Using some examples, we consider how the values calculated for the conductive thermal conductivity by the above method agree with those from the method given in [2].

From Eq. (3) and the assumption that the material is isotropic and contains fibers of one type and that the fiber diameter is constant and much smaller than the length we rather easily obtain an expression with virtually the same form as that given in [2], but the differences in λ_{fr} increase as the assumptions adopted in [2] are violated, i.e., as the structure becomes more inhomogeneous. An increase in the anisotropy of the structure, therefore, results in a lower conductive thermal conductivity, and the decrease may be substantial. Figure 4a shows the results of calculations of the dependence of the conductive thermal conductivity on the relative pressure P at 1100 K for materials based on quartz fiber with a different structural anisotroy (density of the material ρ_{fr} = 150 kg/m^3, fiber diameter $2\cdot10^{-6}$ m; the effect of the finite fiber length was not taken into account).

Since fibers in composites have elongations, which vary over a wide range of values, the degree of the effect of the fiber length on λ_{fr} is an important problem. From an analysis of the calculated results it follows that a a shortening of the fibers causes the number of defective cells to increase and λ_{fr} to decrease. At elongations of more than 200 the fact that the fiber length is finite may be disregarded. Figure 4b shows the results of calculations of the conductive thermal conuctivity of a quartz ceramic with different fiber lengths (T = 1100 K, ρ_{fr} = 150 kg/m^3, d = $2\cdot10^{-6}$, and the structure is isotropic).

Studies on the effect that the distribution of the fiber size has on the conductive thermal conductivity is of the greatest interest. Figure 4c shows the results of calculations of λ_{fr} for a quartz ceramic (ρ_{fr} = 100 kg/m^3) formed by fibers of two types. The first group of fibers has the dimensions $d_1 = 2\cdot10^{-6}$ m, $\ell_{f_1} = 100\cdot10^{-6}$ m and $d_2 = 20\cdot10^{-6}$ m, $\ell_{f_2} = 400\cdot10^{-6}$ m. Fibers of different groups have the same probability of appearing. Comparison of λ_{fr}, obtained by the method described above (curve 2), with the values of the conductive thermal conductivity, calculated from the average fiber diameter (d_{av} = 11 μm, curve 1) reveals a difference that depends on the pressure in the gas medium.

The above calculated results permit the following conclusions:

a) disregard of the size distribution of the fibers and the use of a determinate model structure can result in substantial errors in the calculation of the conductive thermal conductivity of fibrous materials in transient pressure regimes;

b) with a statistical approach to the modeling the effect of a broad spectrum of structural anomalies of the material on its thermophysical parameters can be taken into account effectively from the same positions.

The proposed method can be extended in a natural way to modeling of the optical characteristics of fibrous composites as well as to modeling of the effect that structural macrodefects having their own peculiar distributions with respect to size and physical properties have on the physical characteristics.

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THE THEORY OF THE RHEOLOGICAL PROPERTIES OF DISPERSE SYSTEMS

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UDC 539.37:532.135

The effective rheological characteristics of stacking identical viscoelastic spheres in a matrix of another viscoelastic material are estimated by methods of ensemble averaging theory.

The intensive development of technological processes utilizing stacks of fine particles as working bodies requires the development of physicomathematical models that permit relating the macrorheological properties of such systems to the singularities of their configuration can be given within the framework of the continual approximation, when the disperse mixture is considered as a homogeneous continuum whose behavior is described by the methods of the mechanics of continuous media. However, even in this case the problem that has still not been solved by far arises of calculating the effective charcteristics of a heterogeneous material as a function of the properties of its phases or components and the singularities of their arrangement.

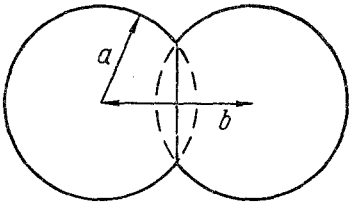

Fig. 1. Model of the contacts between spheres. Dashes are the geometric surfaces of continuation of the spheres.

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